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Nurturing young student mathematicians

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Abstract

Developing mathematical talent in our students should be of primary consideration in education today as nations respond to the challenges of economic crises and ever-changing technological advances. This paper describes two U.S. federally funded curriculum projects, Project M^3, Mentoring Mathematical Minds, and Project M^2, Mentoring Young Mathematicians for students ages 5 through 12. These projects foster in-depth understanding of advanced mathematical concepts by challenging and motivating students to solve and discuss high-level problems in a fashion similar to practicing mathematicians. The curricula have undergone national field tests with proven research results showing significant achievement gains for students studying the curricula over a comparison group of like-ability students. This paper outlines the philosophy behind each program and its connection to the literature and best practices in the fields of gifted education and mathematics education. Next, specific instructional strategies integral to both curricula are outlined. These strategies help teachers establish a community of learners that promotes rich discussions as a platform for posing and solving interesting problems, constructing viable arguments, and defending as well as critiquing solutions. Finally, strategies to help young student mathematicians develop clear and logical written justifications for their mathematical reasoning and share their creative insights are described.

Keywords

Mathematics, gifted curriculum, elementary education, research-based curriculum, in-depth understanding, communication, learning environment

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> Outstanding mathematical ability is a precious societal resource, sorely needed to maintain leadership in a technological world.
>
> — (NCTM, 1980: 18)

The following question was posed to a group of Project M^3 students:

> Miranda thinks all squares are rectangles. Do you agree or disagree with her thinking? Explain your thinking.

> Jacinta wrote:[sic]

> I agree to Miranda's theory. I agree because a square has all the atributes of a rectangle. Those atribiutes are: 4 sides, 4 90° angles, and *2 sets* of oppisite parellel and congruent lines. A square fits all those atributes but it also has 1 extra atribute. That all its sides are congruent. A square also has many other names. Those are: rectangle, parallelogram, rhombus, and quadrilateral. But its clearest name is square.
>
> — (Gavin et al., 2007a: 20)

Reading this at first glance, you may be impressed with the quality and thoroughness of the justification. The abundant and appropriate use of mathematical terms may also have stood out. It might have surprised you how the student was able to express her thinking in writing. It is certainly evident that this student understands that geometric shapes can be classified hierarchically (e.g., squares have one more defining property than rectangles – mainly, the sides of a square are all congruent.) Many students do not understand that individual shapes, much less the relationships among shapes, are categorized by their properties until their high school years (ages 13–16) (Clements, 2003). So, what may be even more impressive about this response is that a fourth-grade 9-year-old student was the author.

This student was participating in Project M^3: Mentoring Mathematical Minds, a United States Department of Education research grant program in which curriculum units for mathematically talented students in grades 3–5 were developed. This curriculum was written to challenge and motivate young students by engaging them in the types of thinking, discussing and writing done by practicing mathematicians. The exemplar response above is a concrete representation of the mathematical reasoning developed across 4 years of field-testing in 61 classrooms in diverse urban and suburban schools. Talented students studying the Project M^3 units significantly outperformed their peers of like ability on open-response questions taken from released items on the Trends in International Math and Science Study (TIMSS, 1994) and the National Assessment of Educational Progress (NAEP; National Center for Education Statistics, 1996). These items were designed to assess in-depth understanding of algebra, data, geometry, measurement, number and probability concepts. (Results show highly significant differences favoring the Project M^3 students with a consistent p-value of < 0.001 for two cohorts of students participating over 3 years. Effect sizes ranged from 0.69 to 1.78. For further description of the research study and results see Gavin et al., 2009.)

The Project M^3 curriculum units foster rich mathematical thinking. This type of thinking, along with the underlying philosophy of encouraging students to think and act like practicing mathematicians, is the basis for a second advanced curriculum project for primary students in kindergarten, first and second grades being developed under the auspices of a National Science Foundation grant, Project M^2. This study is currently in progress and results will be forthcoming (Gavin et al., in press). It is noteworthy that we are finding promising results in terms of achievement gains similar to Project M^3. The aim of both projects is to have elementary students think in depth about challenging mathematical concepts, as mathematicians do, and to make their thinking more public and accessible to the entire class with the use of verbal and written communication. The results are a classroom transformed into a true mathematical community of sharing that result in written responses such as Jacinta's.

This article describes ways in which both curriculum projects help teachers develop an innovative and unique learning environment for elementary students where students are challenged to think and act like young mathematicians. We next present some of the literature that informed the development of the units followed by a description of the instructional strategies and learning environment that emerged as part of the curriculum projects.

Background

George Polya, a well-respected mathematician, believed that the only difference between the work of a professional mathematician and a talented student of mathematics was in the degree of sophistication they use (as cited in Sriraman, 2008). Polya believed that students are capable of mathematical creativity just as mathematicians are, with each operating at their own level of understanding. The philosophy of both Project M^3 and Project M^2 builds on this and is grounded in gifted education pedagogy that focuses on students working and learning in the same way that practicing professionals in the field do. In particular, our underlying philosophy of student as practicing mathematician was based on the Multiple Menu Model for curriculum design developed by Joseph Renzulli (Renzulli et al., 2000). This model promotes the creation of instructional activities that engage students in exploring key ideas that are akin to a particular field of study. Students are encouraged to use the same investigative methods that practitioners in the field do to seek answers to their questions and make contributions to their field. The Curriculum of Practice from the *Parallel Curriculum Model: A Design to Develop High Potential and Challenge High-Ability Learners* (Tomlinson et al., 2009) was also embedded in the units. The Parallel Curriculum Model was written by leading experts in the field of gifted education to guide curriculum developers in their quest to produce high-quality, challenging materials for gifted students. The Curriculum of Practice is one of four types of curricula outlined in the Parallel Curriculum Model. Using the Multiple Menu Model as its foundation, the Curriculum of Practice delineates two functions for the student mathematician as learner. First, as a scholar, the young mathematician uses similar knowledge, problem-solving strategies, and mathematical tools that a mathematician would use to develop a deeper understanding of the mathematics being explored.

Second, as an expert practitioner, the student uses the same methods to produce new knowledge, that is, create something original.

In order to help students acquire the skills of the practicing professional, the materials with which they work must be rich with problems and situations similar to what a practitioner would encounter, albeit at the appropriate student level. Thus, the curriculum content for Projects M^3 and M^2 has as its basis the Core Curriculum, another of the curricula in the Parallel Curriculum Model (Tomlinson et al., 2009). According to this model, the Core Curriculum "is the foundational curriculum that should establish a rich framework of knowledge, understanding, and skills most relevant to the discipline." This curriculum should cause students to "grapple with ideas and questions, using both critical and creative thinking" and should be "mentally and affectively engaging and satisfying to learners" (p. 21). Using the design structure of the Core Curriculum model, the authors based their curriculum on the essential mathematical concepts and processes outlined in the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* (2000) and the *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (2006). In doing so, the curriculum aligns with the new Common Core State Standards (CCSS) for Mathematics (Council of Chief State School Officers & National Governors Association, 2010). The CCSS have recently been adopted at the mathematics framework for grades K-12 curriculum by 46 of the 50 states in the U.S.

From theory to practice: a classroom of young student mathematicians

Before we explore what a classroom of student mathematicians following these tenets might look like, we begin by taking a look at typical elementary U.S. mathematics instruction today. It is actually easier to compare typical classrooms with what mathematicians do not do than with how they actually practice. So what don't mathematicians do? Mathematicians do not start with a problem to which they already know the answer, or even one that they immediately know how to solve. These are simply not interesting. They do not have someone at the front of the room telling them how to solve a problem and then engage in doing 20 more of the same kind of problem. Again, this is not interesting to them. Yet, most elementary mathematics curricula are designed in this fashion and/or delivered in this manner. Rarely does one find a long discussion involving the entire class centered on agreeing and disagreeing with conceptual thinking. Rarely does one find students tested with challenging problems that interest them so much so that the problem-solving experience goes beyond a class period without a solution being found or told. This is so different from the real work of practicing mathematicians.

What *do* mathematicians do? They love to grapple with problems in which they may have no idea of even where to start. They just dig in and start trying some strategies. They are not afraid to change strategies and direct their thinking elsewhere when a solution is not forthcoming. They try to find connections between the problem and other areas of mathematics and/or real-life situations. Unlike the stereotypical picture of the solitary mathematician working behind closed doors, they talk to each other. In doing so, they come up with new, "outside of the box" ideas to try. They persist in solving a problem until that "a-ha" moment arrives. It could take days, months, even years! But when it does arrive, it is infinitely satisfying. Then, they usually write about how they solved the problem and focus on making the explanation clear to themselves and, in so doing, clear to others. They call this an "elegant" solution.

This is what our young mathematicians should do, too. Student mathematicians need to enjoy problems that are challenging; ones in which they might not know where to begin. Student mathematicians need to struggle with a problem, try out a variety of strategies, talk to fellow classmates and their teacher in trying to solve it, and find new ways to solve it. Student mathematicians need to, and want to, persist in problem solving until that wonderful "a-ha" moment arrives. Student mathematicians need to talk about their reasoning and listen to others' explanations. In doing so, a deeper understanding of the mathematics emerges. Student mathematicians need to write about their reasoning to convince themselves and to convince others. Student mathematicians need to discover the joy in creating new problems to solve and the ultimate joy in solving those problems. Most of all, student mathematicians need to love doing mathematics. So, upon entering a Project M^3 or M^2 classroom, one will encounter an environment that promotes this type of learning and love of mathematics. Students are engaged with the mathematics. They struggle to solve problems as they talk and listen to each others' ideas. And they love what they are doing.

Supporting the participation of students as mathematicians

In order to develop a community of student mathematicians mimicking the thinking of professional mathematicians, Projects M^3 and M^2 provide teachers with tools to support students to speak, listen, and write mathematically. We present features across the two projects, including the classroom environment, the verbal model, the nature of the writing tasks, and an instructional tool that helps students connect verbal ideas with their writing.

Classroom environment

Early on, we recognized the need to set up a nurturing environment where all student mathematicians' ideas are considered important. Furthermore, we wanted students to make sense of those ideas and incorporate them into their own thought processes. If you were to visit a Project M^3 or M^2 class, you would not see students raising their hands wildly while someone else was speaking or looking out the window lost in thought, even if those thoughts are mathematical in nature. What you would see is a respectful environment that promotes a community of thinkers and problem solvers. In order to establish such classroom norms, we provided students with guidelines about how to participate in classroom discussions.

The Rights and Obligations incorporated into the Project M^3 units (Gavin et al., 2007b) (Figure 1) helped define learning expectations in terms of what was valued in the environment and establish a supportive culture in which students were encouraged to take risks, try new strategies, and ask questions when uncertain. Teachers took note of

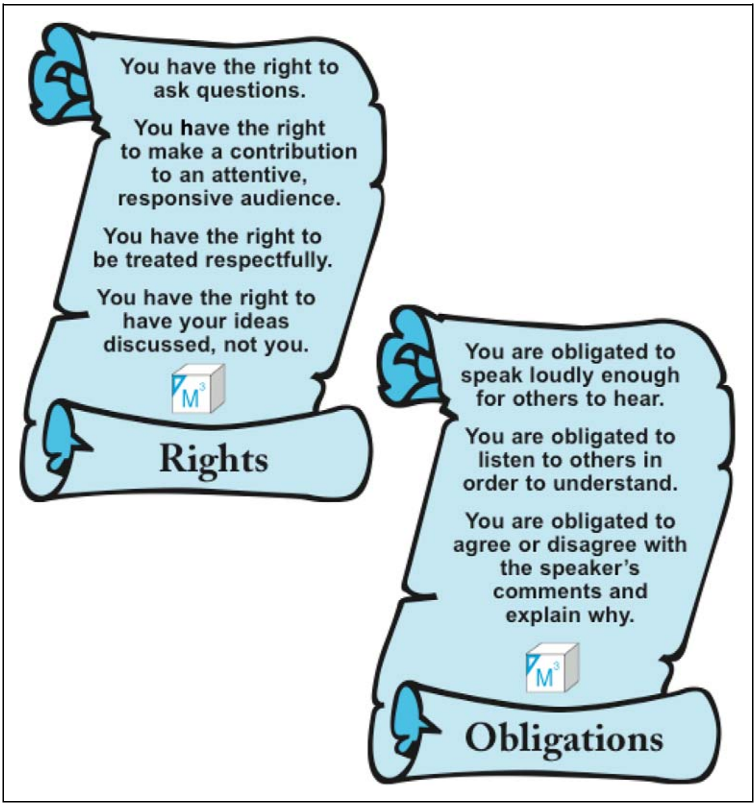

Figure 1. Rights and Obligations used in the Project M^3 units (Gavin et al., 2007b). From *Student's Mathematician Journal, Project M^3: Mentoring Mathematical Minds*. Adapted from Chapin, S.H. (1998–2002) *Project Challenge: Identifying and Developing Talent in Mathematics within Low-income Urban Schools* (Jacob K. Javits Gifted and Talented Students Education Act Grant No. R206A980001). Washington, DC: US Department of Education. Reprinted with permission from Kendall Hunt Publishing.

how the Rights and Obligations prompted students to take greater responsibility for their own learning and to respect all students and their ideas. Students were encouraged to focus on the conversation and ideas being shared rather than the person sharing the ideas. In this way disagreements never got personal but were honored as a way to better understand the mathematics.

Kristen, a fifth-grade teacher, reflected on how the Rights and Obligations impacted her instruction: “As a group, students view themselves as a community of mathematicians. I think this sense of community has been especially powerful.” Jack, a fourth-grade teacher described how the learning environment positively impacted one of his students: “The most important success that he has had this year . . . is that he has become more comfortable discussing mathematical ideas. He has realized that his ideas have meaning and that others are interested in what he has to say.”

To acknowledge the need to be more explicit with younger students in kindergarten, first and second grades, the Project M^2 units represented these Rights and Obligations as ways in which students should behave as both speakers and listeners (Gavin et al., 2010a). All of these roles give credence to students' ideas. The *speaker roles* ensure that students:

- speak loudly enough to be heard;
- relay their thoughts to the class, not just the teacher;
- explain their ideas so others will understand them; and
- agree and disagree with others' ideas rather than with the person.

Not only is it important to contribute to class discussions, listening to what fellow student mathematicians say is equally important. Strong listening skills are actually more difficult for students to develop. They are generally more eager to share their own ideas rather than hear what others have to say. We found this is especially true with talented, creative students who have many new ideas that they wish to contribute. However, listening to others' ideas helps them evaluate their own idea and will benefit them greatly in collaborative problem solving as members of the workforce. The *listener roles* encourage students to:

- ask the speaker to speak up, if necessary;
- demonstrate that they are listening (i.e., their bodies are positioned in a way to show they are listening) and making sense of ideas; and
- ask questions when needed to clarify thoughts.

The speaker and listener roles go hand in hand and help support an environment that nurtures students as mathematicians. Our best indicator of success in nurturing this environment was the frequent comment from students after listening and making sense of others' ideas: "I now disagree with myself!"

Verbal communication

Facilitating discussions. For two decades, the NCTM has been calling for teachers to move away from talk that is more didactic (where the teacher acts as the knowledge bearer and students as repositories of this knowledge) towards one that positions students as part of a sense-making community. This has major implications for the nature of discussions, also commonly referred to as discourse. Discussions should center "on mathematical reasoning and evidence as the basis for the discourse. In order for students to develop the ability to formulate problems, to explore, conjecture, and reason logically, to evaluate whether something makes sense, classroom discourse must be founded on mathematical evidence" (NCTM, 1991: 34). Students in these classrooms engage with fellow peers to make sense of the mathematics, with the teacher acting as a facilitator of such interactions (NCTM, 1991). Our vision of student mathematicians incorporates such exchanges. Nevertheless, orchestrating this discussion can be challenging, and, in fact, one of the reasons for NCTM's 2000 publication, *Principles and Standards for*

School Mathematics, was that discourse was not being implemented as intended (NCTM, 2000).

> Teachers are expected to facilitate discussion so that students:
>
> - "organize and consolidate their thinking through communication;
> - communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
> - analyze and evaluate the mathematical thinking and strategies of others;
> - use the language of mathematics to express mathematical ideas precisely"
>
> — (NCTM, 2000: 60)

Although teachers may agree with these expectations and understand *what* the talk should look like, we felt it necessary to incorporate tools that would guide teachers as to *how* to implement such discussions. This is especially important when the content is advanced and the goal is to foster high-end learning in order to nurture talent. As a result, we adapted Chapin et al.'s (2009) "talk moves." Although at first glance they may appear to be simplistic, these talk moves provide teachers with the tools to orchestrate discussions that help ensure that all students understand the questions being posed, allow them to grapple with and make sense of the mathematics, and come to mathematically valid conclusions. All the while, teachers encourage students to interact with others' ideas, much like professional mathematicians do. The talk moves (Gavin et al., 2010b) include:

- repeat and check;
- agree/disagree and why;
- partner talk;
- add on; and
- think time.

Repeat and check has students or the teacher repeat an idea shared by a student. Then the teacher confirms that the repeated idea was heard accurately. For example, a teacher can ask, "Joshua, can you repeat what Emma said? . . . Joshua, is that what you meant to say?" This talk move serves to set students' ideas as the center for discussion, help students clarify their thinking, and allow others more time to consider an idea. Once the topic of the discussion has been established and the teacher feels students have had sufficient time to digest it, she can move on to *agree/disagree and why* where students reason mathematically about the given idea: "Do you agree or disagree with Tristan's idea? Tell us why." This talk move places the onus of defending the mathematical validity of answers on students—in a similar vein as professional mathematicians—rather than the teacher. *Partner talk* is a talk move designed to give students an immediate audience to grapple with their ideas and make their thoughts more cohesive and clear. *Add on* prompts students for further participation, which serves to open up the discussion and elaborate on others' ideas. Lastly, *think time* can be used throughout the discussion to allow students to formulate their thoughts before and after they are called on to share their ideas with the group. It demonstrates that student reasoning is valued and encourages all students to participate. The following sample dialogue connected to the question posed to students at the beginning of this article incorporates the talk moves to help student mathematicians reason about whether or not all squares are rectangles.

Teacher: So do you agree with Miranda? Is a square a rectangle, too? Why or why not? Jackie.

Jackie: I think she is wrong. A square looks different than a rectangle. The two rectangle sides are long, but the square ones are not.

Teacher: Did you say that a square is not a rectangle because the sides of a rectangle are long and the sides of a square are not?

Jackie: Yes. Squares look more like a box.

Teacher: Who agrees or disagrees with Jackie's idea and can tell us why? Scott?

Scott: I think I disagree with her idea because squares have things that are the same as rectangles.

Teacher: Leena, can you add on to what Scott just said?

Leena: I know that squares and rectangles both have four sides.

Teacher: Who can repeat what Leena just said? Edya?

Edya: I think she said that squares are rectangles. They both have four sides.

Teacher: Turn to your partner and talk about what else might be the same about squares and rectangles. [Partners discuss for about 2 minutes.] Lenny, what did you and your partner talk about?

Lenny: We said that both squares and rectangles have square corners.

Teacher: Who can add on to this idea? Maxie?

Maxie: I think that they mean they both have all 90° angles.

Teacher: So I hear you saying that the squares and rectangles both have four sides and have all 90° angles. Richard?

Richard: Tommy and I said that the square sides have to be the same.

Teacher: Do you mean the sides of the square are the same length? Do you have a math vocabulary word that could be used here?

Richard: Yeah. They are not long, just the same. Congruent.

Teacher: Now talk to your partner about these ideas. Is a shape with four sides that are the same length and has four 90° angles a rectangle? [Students talk for about 3 minutes.] Henry, what do you and Isabella think, and why?

Henry: We think so because a rectangle has to have four sides and four 90° angles—a square has all of this! It just has one more thing. Cause, well, it's special. It has, um, the sides are all the same.

Teacher: Can someone repeat what Henry said? Gina?

Gina: Henry said that a square has to be a rectangle because it fits what a rectangle means. It's just that a square has an extra thing about it—the sides are congruent.

Developing mathematical vocabulary. As the previous dialogue indicates, it is essential for students to incorporate mathematical vocabulary into their reasoning so that other student mathematicians can better understand the message: "It is important to give students experiences that help them appreciate the power and precision of mathematical language" (NCTM, 2000: 63). The Project M^3 and M^2 units provide additional support

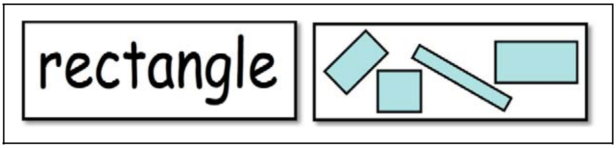

Figure 2. Sample word wall card set. Reprinted with permission from Gavin et al. (2010b) © 2010 Kendall Hunt Publishing Company.

in this area, including providing teachers with a mathematical language section in each lesson, a teacher and student glossary of terms, and a word wall. The mathematical language section within each lesson includes the list and definitions of vocabulary that teachers can anticipate students will use during the lesson, making note that students are not expected to regurgitate a formal definition.

In the Project M^2 units, we have designed the student glossary (Gavin et al., 2010a) to be interactive in nature for these younger students. This glossary in the back of their Student Mathematician's Journal contains several pictorial representations of important vocabulary words. When the teacher introduces the vocabulary term, the student finds the pictures that match the term and writes the word in the blank space next to the term. The word wall mimics the student glossary. Each term includes one card with the name and another card with several representations (Figure 2). Teachers are encouraged to post the word walls in a prominent location and use them during instruction. They might ask students to repeat or add on to someone's idea using a word from the word wall. Students also can interact with the word wall by playing matching and sorting games. Thus, our student mathematicians are actively engaged in developing meaning for mathematical vocabulary and using it appropriately in their discussions and writing.

Talk frame

The talk frame was infused into the units to serve as a vehicle that connects verbal and written communication. It is a graphic organizer used on the board that captures student ideas about a significant and high-level mathematical question as it unfolds during a discussion (Gavin et al., 2010b; Williams and Casa, 2011/2012). Casa (2012) explains that the talk frame (Figure 3) begins with the "Think" section that has students reword the question to ensure that they understand what is being asked of them. The teacher paraphrases all ideas shared by students under multiple "Talk Ideas," and these include correct mathematical ideas as well as underdeveloped ones and misconceptions. This feature forces students to rely on their reasoning rather than the teacher's affirmation to determine the mathematical validity of ideas—similar to how professional mathematicians work to solve problems. An impetus of the talk frame was to capture student ideas to give them more permanence than just the spoken word. This allows peers to revisit previously shared thoughts and build upon them. Finally, when the class reaches a mathematically valid conclusion, the teacher records students' summaries of their understanding in the "We Understand" section. Figure 3 presents a sample of a talk frame that would capture the discussion presented in the previous dialogue. Note that students would have discussed what was being asked of them. This rewording of the question

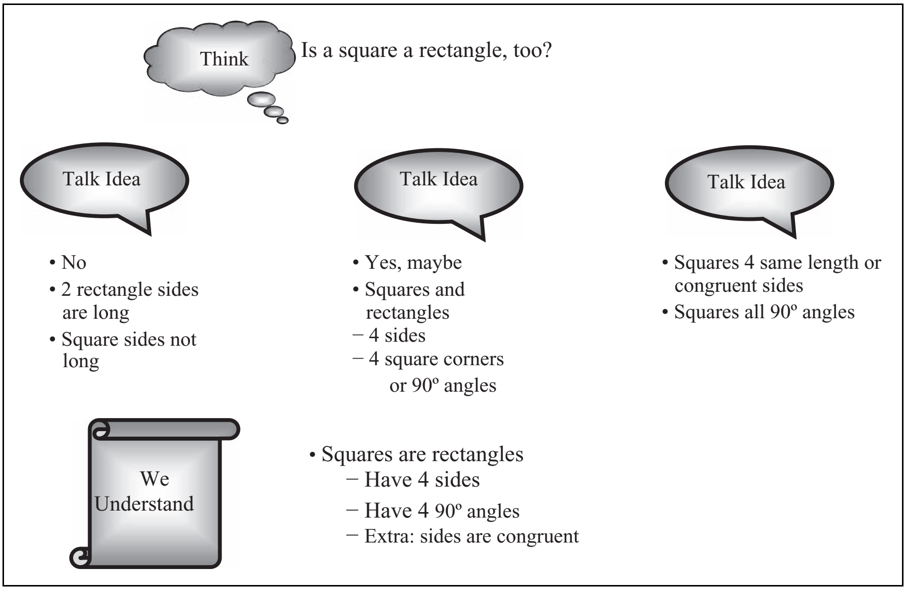

Figure 3. Sample talk frame representing a discussion about whether or not squares are rectangles. General talk frame © 2010 Tutita M. Casa.

is just one example of how this could be done, as are the contributions made by students and how a teacher records them. Regardless, the "We Understand" summary would represent the same valid mathematical conclusion.

Written communication

Although NCTM notes that "written communication should be nurtured" (2000: 62), there is little guidance about how to go about this. In the Project M^3 and M^2 units, the talk moves and talk frame serve to provide a model to help develop quality written responses to "Think Deeply" questions. These high-level questions, posed at the end of each lesson, typically take 3 days to resolve and are focused on a significant mathematical idea from the lesson, such as the Think Deeply question offered at the beginning of the paper. In fact, we consider the Think Deeply question the heart and soul of the lesson.

As a collection, the Think Deeply questions encourage students to reason about "sound and significant mathematics" (NCTM, 1991: 25). To help them learn the conventions of quality mathematical writing, we approached this in a similar fashion as students learning to write in any other genre (NCTM, 2000), and considered what professional mathematicians would require of one another. We developed writer's roles that include:

- thinking about the question;
- talking about the answer; and
- telling all ideas, the answer, and why.

We also encourage teachers to scaffold the introduction of mathematical writing into their teaching. To begin, students complete the initial Think Deeply question as a class while discussing the characteristics of the writing. Then partners write some responses together for the next couple of Think Deeply questions. Finally, individuals compose their own response for the remainder of the Think Deeply questions in the unit. Throughout, students share their work to give peers the opportunity to see that "writing" can include not only words, phrases, and sentences, but also other representations to support those ideas, such as drawings. In a similar fashion to verbal exchanges, students see a reason to use mathematically precise vocabulary in an effort to more effectively communicate their ideas (NCTM, 2000).

Conclusion

We have found that with a mathematical learning community established using the structure, instructional strategies, and curriculum described above, students have made great gains in terms of deep mathematical understandings of advanced concepts, as evidenced in our research results. Just as rewarding, we have found that students truly love mathematics and describe math class as "being in heaven." When asked what he wanted to be when he grew up, one first grader wrote that he wants to grow up to be a "Mathematician Texan!" We believe that encouraging this passion for and understanding of mathematics at a young age is an essential component in developing future career mathematicians. We must nurture the talents of our budding mathematicians in order to create a global society where the workforce is capable of innovative mathematical problem solving. And we can never begin this process too early.

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Biographies

Dr. M. Katherine Gavin is an associate professor at the University of Connecticut where she serves as the mathematics specialist at the Neag Center for Gifted Education and Talent Development. She is the director and senior author of two multi-year curriculum research projects that involve the development of advanced mathematics units for mathematically talented elementary students from kindergarten to fifth grade. Project M^3

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Dr. Tutita M. Casa is an Assistant Professor in the Department of Educational Psychology at the University of Connecticut and is the co-principal investigator of Project M^2. Her primary interest and research focuses on supporting teachers in the implementation of high-level mathematical discourse. She has carried out this goal in numerous ways, including being a unit co-author, developing the talk frame, writing publications, and offering numerous presentations at the national level.